ECE 313 Lecture Notes

Study Day / Extended Office Hours

Tuesday, November 3, 2026

By TA Alice Liu

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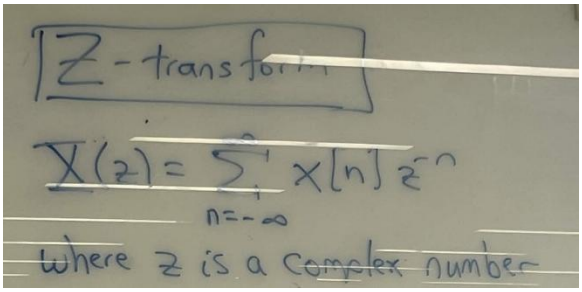
Addressing the following topics…

- Mini-Project #2 Questions
- Midterm #2 Questions/Content

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Student Question: What does the 'z' in the Z-transform polynomial represent?

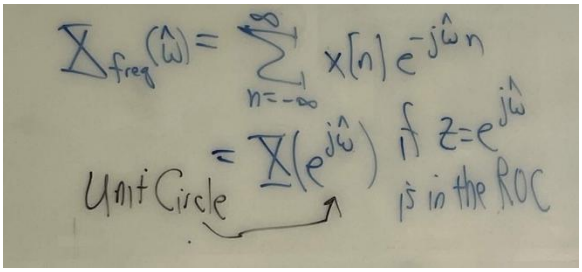
Answer:

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- Trying to find the components of $x[n]$ that correspond to different delays z^{-n}

Student Question: What's the difference between Z-Transform & Frequency Response?

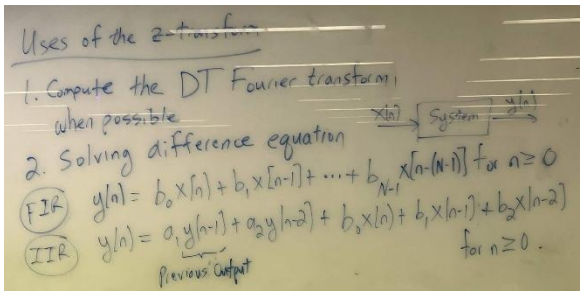
Answer:

- They are connected!
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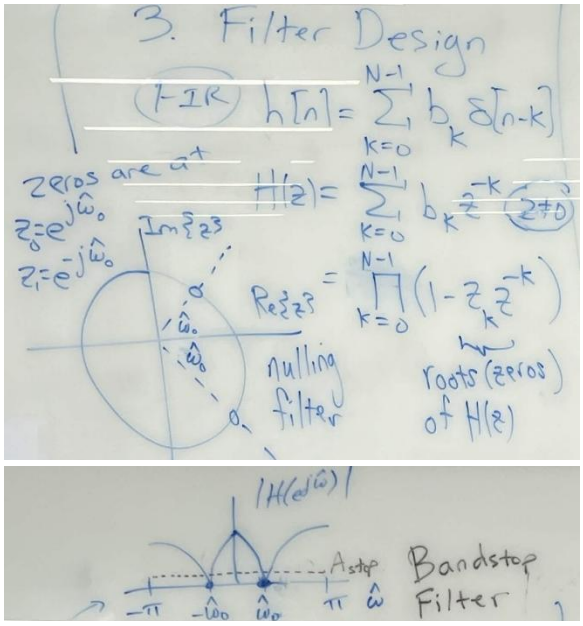
- Frequency Response is the Z-Transform evaluated for $z = e^{j\omega}$
    - IF $z = e^{-j\omega}$ is in the Region of Convergence (R.O.C.) for the transform

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Uses of the Z-Transform:

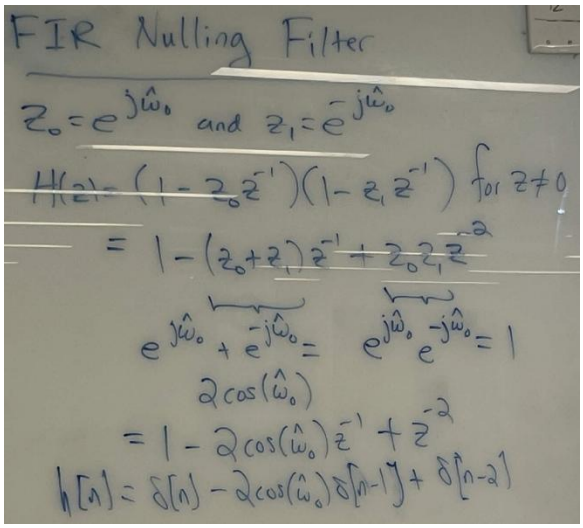


- (2. Continued…) Solving the Difference Equation:
  - We represent it as a system between input & output
  - System is NOT necessarily LTI
    - If the Initial Conditions are zero, the system is LTI
  - Even if the system is NOT LTI, we can still use the Z-Transform to analyze it
    - For Analysis: A way to find the closed-form solution for a formula for the output $y[n]$ given the input $x[n]$ esp. for IIR systems with feedback

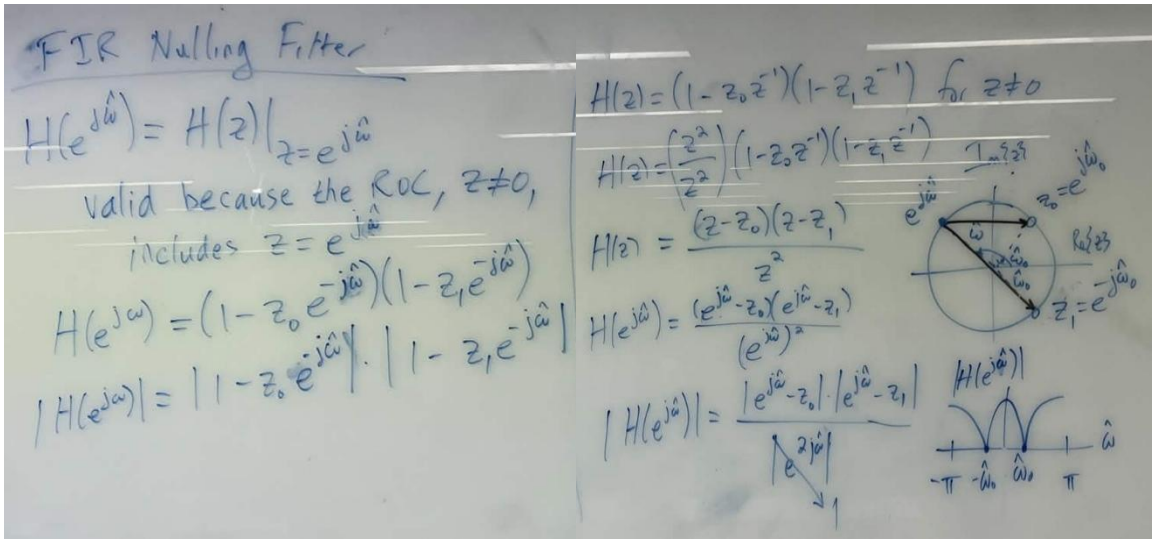


- (3. Continued…)
  - For FIR filter design, we can compute the transfer function as a polynomial in z^{-1} and find its roots (zeros)
  - And the polynomial roots tell us…
    - What frequency selectivity the filter has -- Lowpass, Highpass, Bandpass, Bandstop, etc.
    - For a zero located on the unit circle, the angle of the zero indicates the DT frequency that is “nulled” or “zeroed” out

- FIR Nulling Filter
  - Wherever you want to zero-out/null out/remove a frequency component, simply put a zero on the unit circle at that angle!



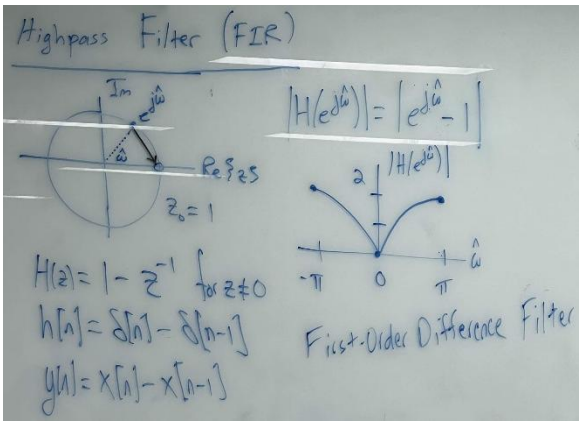
  - Why is it that the roots of the polynomial (a.k.a. the "zeros") null out frequency components? Let's take a look at deriving the magnitude response and seeing it's relation with the polynomial roots ("zeros")



- In essence: the zeros control the magnitude response, which can be seen when we appropriately convert $H(z)$ to $H(e^{j\omega})$
- Thus: wherever I put a zero on the unit circle, its angle will be correspondingly zeroed out in the magnitude response

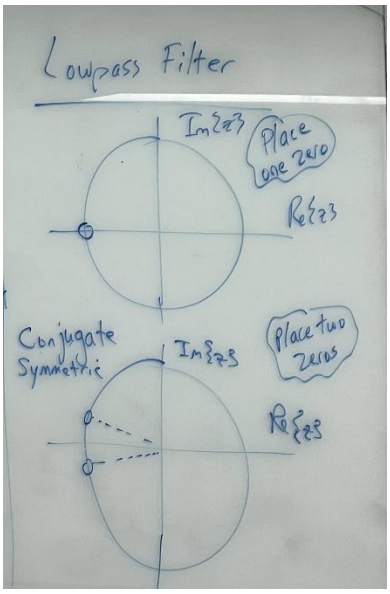
- Side-question: Where does the factored form in $H(z)$ or $H(e^{j\omega})$ come from?
  - When analyzing a difference equation; you must compute the transfer function $H(z)$and derive the factored form from the resulting polynomial (factor the polynomial)
  - When having a transfer function $H(z)$ in polynomial form, you must factor the polynomial
  - Sometimes, the transfer function $H(z)$ is already in factored form
  - Note: For $H(e^{j\omega})$, you may simply convert it from $H(z)$, provided it the unit circle $z = e^{j\omega}$ is the Region of Convergence (R.O.C.)
    - Specifying/proving it satisfies R.O.C. is justification to convert from $H(z)$ to $H(e^{j\omega})$

FIR Filters:



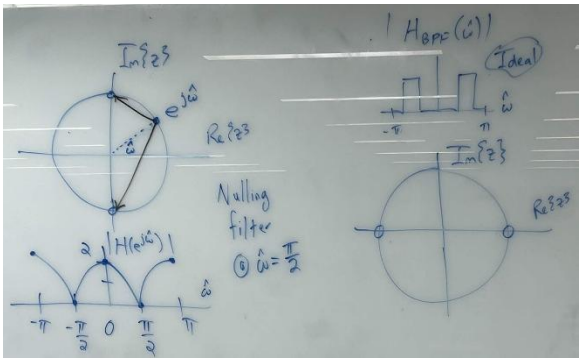
<u>High-pass Filter (FIR)</u>

- 1 zero at $z_0 = 1$
- Zeros out frequencies around that zero
  - Since the zero has an angle of $0^{*}\pi\ radians$, all the lower frequencies are zeroed out, leaving only the high frequencies



<u>Low-pass Filter (FIR)</u>

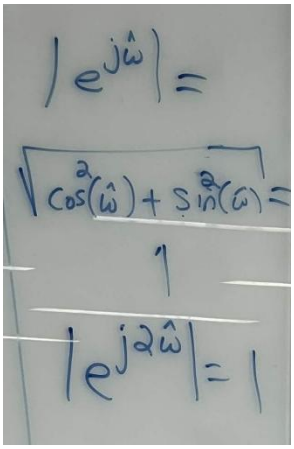
- Upper Design: 2 zeros at the same angle ($\pi\ rad$)
- Lower Design: 2 zeros in the high frequencies, with conjugate symmetry
  - Conjugate symmetry to have real-valued filter & output



Band-pass Filter (FIR)

- Zeros between high & low frequency angles

Student Question: How to relate magnitude of $e^{j\hat{\omega}}$ to value of 1, that is: $\left|e^{j\hat{\omega}}\right| = 1$



- Remember: in polar form, the magnitude of $e^{j\hat{\omega}}$ is 1
- Or we can rewrite $e^{j\hat{\omega}}$ in terms of cosine & sine equations using Euler's identity, and compute the magnitude as usual, which is 1.